Project 3… Estimating Distance to the Moon from its Speed - 1

Step 1: Determine the surface speed of the shadow between two points along the path of totality.

Example: Newport, Oregon to Madras, Oregon is 232 km and the UT mid-point times differ by 0.0619 hours so the derived ground speed is 3746 km/hr.

Step 2: Determine the angle along the surface of Earth between the observing point and McClellanville, South Carolina, **q**, along the path of totality. This can be done using a piece of string and comparing the distance to the circumference of the earth along the same Great Circle, which equals 360 degrees. You can also use this formula:

x = sin(Lat1)sin(Lat2)+cos(Lat1)cos(Lat2)cos(Long1-Long2)

q = arccos(x) in degrees

Example:

Madras Oregon : Latitude (Lat1) = **44.7N** Longitude (Long1) = **121.1 W**
McClellanville, SC : Latitude(Lat2) = **33.05N** Longitude (Long2)= **79.53W**

X = sin(44.7)sin(33.05) + cos(44.7)cos(33.05)cos(121.1-79.53)
X = 0.8293

Q = arccos(0.8085) = **34.0^{o}**

Step 3: Use a formula to predict the ground speed at your location from the speed of the moon in its orbit and corrections for Earth's rotation and its orbit around the sun. The model also corrects for geometric projection effects that distort the speed and shape of the eclipse path.

The angle that the eclipse track makes with a meridional line of longitude is 23 degrees, so the ground speed of the shadow's movement has to be corrected for Earth's west to east rotation along this path by a factor of cosine(23). Also, the Earth's speed at your latitude has to be factored into the calculation. You also have to include the elevation angle of the eclipse above your horizon. The shadow speed that you measure on Earth's surface is related to the lunar distance,**D**, the angular distance to our reference point in McClellanville, SC, **q**, and the elevation of the eclipse above the horizon **a** by the formula:

$$\text{Speed} = 1.23\cos(23^{o})6.61\times10^{-5}\,\frac{(3557 - 1674\cos(\text{Latitude}))\,(151390000)}{(378475 - 1.0025\mathbf{D} + 6378\cos(q))\cos(q)\sin(a)}$$

For example, if you are observing in Madras, Oregon where the latitude is 44.7N, and the eclipse is at an elevation above the horizon of a= 41.6°, and the angle to McClellanville from the example in Step 1 is q=34, then the observed shadow speed will be given by the formula:

Equation 1:

$$\text{Speed} = 1.23\cos(23^{o})6.61\times10^{-5}\,\frac{(3557 - 1674\cos(44.7))\,(151390000)}{[378475 - 1.0025\mathbf{D} + 6378\cos(34)]\,\cos(34)\,\sin(41.6)}$$

Which simplifies to:

$$\text{Speed} = \frac{4.87\times10^{7}}{383762 - 1.0025\mathbf{D}}\ \text{km/hr}$$

From your measured values for your latitude, elevation of the sun at totality, and distance from the reference site in McCLellanville, SC, create an equation similar to the one above after simplifying Equation 1 using your observed values for your latitude, a and q.

Step 4: Compare observed ground speed between neighboring observing sites with predicted model:

Example: In the Newport area we measured the speed as 3746 km/h, so

$$3746 = \frac{4.87\times10^{7}}{383762 - 1.0025D}$$

Step 5: Solve the equation in Step 3 for D, which is the lunar distance in kilometers.

$4.87\times10^{7} = 3746\,(383762 - 1.0025D)$ *Divide both sides by 3746*

$13000 = 383764 - 1.00D$ *Subtract 383764 from both sides*

$-370764 = -1.0025D$ *Divide both sides by -1.0025*

$D = 369839$ km

Round this to three significant figures to get 370,000 km.
Actual value is 372,000 km.