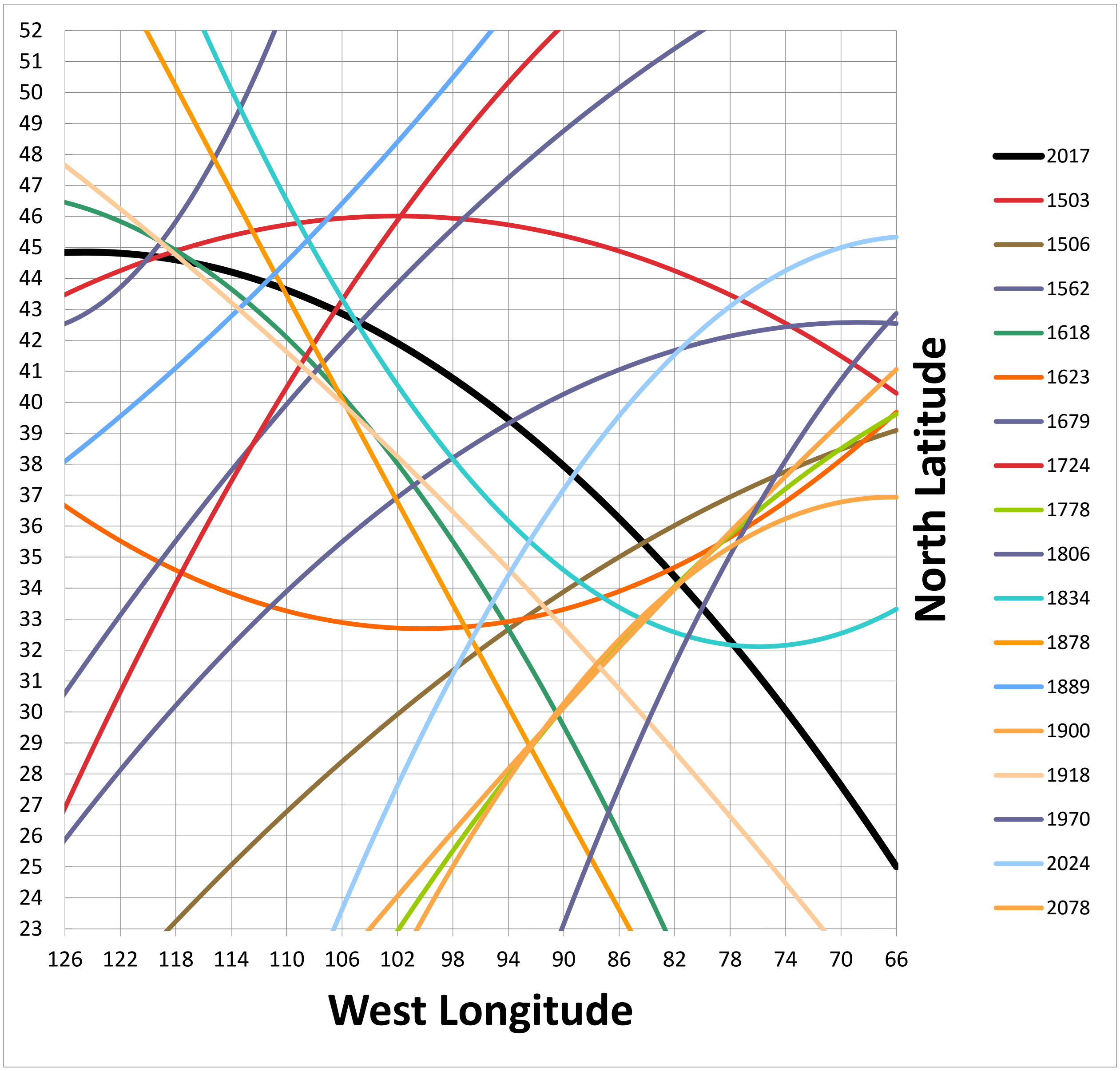
Exploring Quadratic Equations

One interesting feature of the August 21, 2017 total solar eclipse is that its path of totality across the continental US crosses over the paths of earlier and later eclipses.

Each of the crossing points can be determined by fitting a quadratic equation to three or more pairs of points along each track, and then solving the two equations to find the common, intersection, point.



Each track has been represented by its own equation in Table 1, so the crossing points can be found by simply setting one equation equal to the other and solving for the value of x.

Date	Fitting function
August 21, 2017	$y = -0.005792x^2 + 1.4425x - 44.968$
March 27, 1503	$y = -0.00441x^2 + 0.8998x + 0.1113$
July 20, 1506	$y = -0.003139x^2 + 0.2726x + 34.776$
February 3, 1562	$y = 0.029785x^2 - 7.6792x + 537.25$
July 21, 1618	$y = -0.009871x^2 + 2.6021x - 124.7$
October 23, 1623	$y = 0.005971x^2 - 1.1966x + 92.538$
April 10, 1679	$y = -0.00392x^2 + 0.3425x + 49.689$
May 22, 1724	$y = -0.00729x^2 + 0.8719x + 32.78$
June 24, 1778	$y = -0.005793x^2 + 0.5098x + 31.2$
June 16, 1806	$y = -0.00508x^2 + 0.6976x + 18.627$
November 30, 1834	$y = 0.01238x^2 - 1.8789x + 103.4$
July 29, 1878	$y = 0.000486x^2 + 0.7316x - 42.9$
January 1, 1889	$y = 0.00328x^2 - 1.1774x + 134.37$
May 28, 1900	$y = -0.00147x^2 - 0.2253x + 62.33$
June 8, 1918	$y = -0.00192x^2 + 0.8297x - 26.41$
March 7, 1970	$y = -0.0142 x^2 + 1.394x + 12.72$
April 8, 2024	$y = -0.012722x^2 + 1.6454x - 7.8473$
May 11, 2078	$y = -0.0119x^2 + 1.5792x - 15.458$

Example:

2017: $y = -0.005792x^2 + 1.4425x - 44.968$
1503: $y = -0.00441x^2 + 0.8998x + 0.1113$

Then $-0.005792x^2 + 1.4425x - 44.968 = -0.00441x^2 + 0.8998x + 0.1113$, which simplifies to: $0 = 0.001382 x^2 - 0.5427x + 45.0793$. The two roots to this quadratic equation can be found using the Quadratic Formula

where $a = +0.001382$, $b = -0.5427$ and $c = 45.0793$

then $x = 196.3459 \pm 77.0248$

so root-1 $x = 196.3459 + 77.0248 = 273.37$ and
root-2 $x = 196.3459 - 77.0248 = 119.321$

The second root is within the continental United States so ignore then first root.
Then for x = longitude=119.321, we can use one of the track formulae to determine the latitude, y:

$$y = -0.005792(119.321)^2 + 1.4425(119.321) – 44.968 = 44.689.$$

So the exact crossing point is at longitude 119.321 or 119° 19' 15" West
and latitude 44.689 or 44° 41' 20" North.

Problem 1: Select an eclipse and from the tabulated quadratic equations determine the crossing point for the track with the 2017 eclipse.

Problem 2: From the plotted curves, find a crossing point between eclipses that do not involve the August 21, 2017 event.

Problem 3: About how many crossing points are there among the tabulated eclipses within the boundaries of the continental United States for which the longitude range is from 75 West to 125 West, and the latitude range is from 25 North to 44 North?